BEL Frahm Damper System

1 Introduction

The area of vibration control usually considers two system types: vibration dampers and vibration absorbers. Vibration dampers are systems that convert vibrational energy into heat and are thus limited by their ability to reject heat. Vibration absorbers are systems that generate counteracting forces that limit motion at the point of interest in the system by absorbing it in the motion of the absorber. In the ideal absorber there is no heat generated and therefore no heat limitation.

By far the best known vibration absorber is the “Frahm Damper,” invented by H. Frahm in the early 20th century (the dates 1909 and 1911 are both cited for his patent and it is not clear which applies). The theory of this device is described in many places, notably J.P. Den Hartog, *Mechanical Vibrations*, 4th ed., McGraw–Hill, 1956, pp. 87–92. It has been applied in many places and in both translational and torsional forms with great success. There has been one particular application of the Frahm absorber that is of particular interest and is the subject of this note.

The Newington Station is a 400 MW cycling fossil fuel power plant in Newington, NH. To be a cycling plant means that it must go from low to high output power levels at least once (and perhaps more often) each day. This is relatively hard duty for a power plant, the easier duty being to be “base loaded” which means to have a steady load all the time. Cycling duty puts the entire system through severe thermal cycles and associated fatigue and this adversely affects the expected life of the plant. Fatigue of components is a major consideration in the operation of a power plant.

The plant has two induced fans (ID fans) driven by 4500 horsepower, three phase induction motors at 890 rpm (synchronous speed would be 900 rpm, so these motors have only slightly over 1% slip). The fans are mounted on concrete block foundations as shown in the end view figure, and the motors are directly coupled to the fans (not shown in the figure). In the original configuration, the absorbers shown in the figure were not present.

When the plant was put into initial operation, severe vibration were found in the fans. The fans could not be balance in both the hot and cold conditions simultaneously, although hot balancing allowed the system to operate in steady state after that condition was reached. Ramp rates were another matter, however, and this is critical for a cycling plant. Ramp rates above 2 MW/min produced severe vibrations and thus limited the ability of the plant to perform its primary function.

In January, 1976, Brewer Engineering Laboratories, Inc. (BEL), under the direction of Mr. Given A. Brewer, P.E., began work at the plant. They made measurements
resulting in a dynamic model for the fan and foundation system, and they presented three possible alternative for solutions. One of those alternatives was the design and fitting of dynamic vibration absorbers (Frahm dampers) for the fans, and the owner elected to have BEL proceed with this approach. This was successful, and eventually the plant was able to ramp at rates of 20 MW/min without problems.

This project eventually involved not only BEL and Mr. Brewer, but also the noted Prof. S.H. Crandall of MIT and a student, A.T. Guillen, who did an MIT BS thesis on the problem (A.T. Guillen, “Control of Machine Vibration by Tuned Absorber,” MIT, 1980). Guillen’s analysis appears a bit overly complex to this writer (he has included damping in the absorber spring which I think is unnecessary), so a less complicated analysis is presented here. Guillen’s paper is the source for the parameter values used in the analysis given here.

Mr. Brewer was a successful business man, and he publicized his work on this project with a number of articles, including the first one that this writer saw: G.A. Brewer, “Power Plant Fan Vibration – Absorber Systems, Sound and Vibration, April, 1983, pp. 20 – 22, and no doubt a number of others elsewhere. He also received a patent on the system (US Patent 4,150,588, Apr. 24, 1979). Mr. Brewer sold his company, BEL, to Teledyne Engineering Services and then remained on with them in a senior consulting capacity for some time. He died April 26, 1987.
2 Analysis

The motion of concern is the horizontal motion at the bearing level. For this motion, the system may be thought of as having two inertias (before the absorbers are added): the pedestal and the rotor, that are elastically coupled and the pedestal is elastically coupled to ground. When the absorbers are added, if all four of them are acting in phase, as they are intended to act, then this is one additional mass, so that at total of three elastically coupled masses comprises the system model. This model is shown in Figure 2.

![System Model, Including Absorbers, for Horizontal Motion](image)

Figure 2: System Model, Including Absorbers, for Horizontal Motion

The actual construction of the absorbers uses vertical rectangular beams as the spring elements. These beams are cantilevered upwards from the steel pedestal just below the actual bearing. The absorber masses are constructed of flat plates. The majority of the absorber stack is welded together to form a solid mass, but there is provision to add or remove thin plates at the top (these are simply bolted on) to adjust the mass for tuning purposes.

Now consider the system equations of motion:

\[
\begin{align*}
\sum F_1 &= -K_1 x_1 - C_1 \dot{x}_1 + K_2 (x_2 - x_1) + C_2 (\dot{x}_2 - \dot{x}_1) + K_3 (x_3 - x_1) = M_1 \ddot{x}_1 \\
\sum F_2 &= -K_2 (x_2 - x_1) - C_2 (\dot{x}_2 - \dot{x}_1) + F_e = M_2 \ddot{x}_2 \\
\sum F_3 &= -K_3 (x_3 - x_1) = M_3 \ddot{x}_3 
\end{align*}
\]

where the external force on the rotor, \( F_e \), is the force of unbalance, \( F_e = -M_2 e \Omega^2 \cos \Omega t \) where \( e \) = eccentricity. These equations can be recast in matrix form as
The parameters for this system are as follows:

- $M_1 = 8.321 \times 10^4$ kg  
  $C_1 = 2.72 \times 10^6$ N–s/m  
  $K_1 = 1.454 \times 10^9$ N/m
- $M_2 = 2.153 \times 10^4$ kg  
  $C_2 = 4.97 \times 10^4$ N–s/m  
  $K_2 = 2.874 \times 10^8$ N/m
- $M_3 = 7.26 \times 10^3$ kg  
  $K_3 = 6.3035 \times 10^7$ N/m

$N = 890$ rpm

$\Omega = 93.201$ rad/s which corresponds to $freq = 14.833$ Hz.

For purposes of calculation, it is assumed that the eccentricity is $0.5$ mm $= 5 \times 10^{-4}$ m.

It is useful to investigate the response of this system for different operating speeds (different values of $\Omega$), in effect a frequency response analysis. For this purpose, assume that all of the responses are sinusoidal at the excitation frequency,

\[
\begin{align*}
\{x\} &= \{a\} \cos \Omega t + \{b\} \sin \Omega t \\
\{\dot{x}\} &= -\Omega \{a\} \sin \Omega t + \Omega \{b\} \cos \Omega t \\
\{\ddot{x}\} &= -\Omega^2 \{a\} \cos \Omega t - \Omega^2 \{b\} \sin \Omega t
\end{align*}
\]

Now substitute into the differential equation to get
\[ [M] \langle -\Omega^2 \{a\} \cos \Omega t - \Omega^2 \{b\} \sin \Omega t \rangle \\
+ [C] \langle -\Omega \{a\} \sin \Omega t + \Omega \{b\} \cos \Omega t \rangle \\
+ [K] \langle \{a\} \cos \Omega t + \{b\} \sin \Omega t \rangle \\
= M_2 e^{\Omega^2 \cos \Omega t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \]

Now separating sine and cosine terms gives

\text{Cosines:}

\[ \langle [K] - \Omega^2 [M] \rangle \{a\} + \Omega [C] \{b\} = M_2 e^{\Omega^2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \]

\text{Sines:}

\[ -\Omega [C] \{a\} + \langle [K] - \Omega^2 [M] \rangle \{b\} = \{0\} \]

Now this can be written in terms of matrices twice as big by partitioning them so that

\[ \begin{bmatrix} [K] - \Omega^2 [M] & \Omega [C] \\ -\Omega [C] & [K] - \Omega^2 [M] \end{bmatrix} \begin{bmatrix} \{a\} \\ \{b\} \end{bmatrix} = M_2 e^{\Omega^2} \text{col}(0, 1, 0, 0, 0) \]

As long as the \([C]\) matrix is nonzero, the coefficient matrix on the left is nonsingular and can be readily inverted to solve for the \(a\)'s and the \(b\)'s. For the full system, including the absorbers, this is a \(6 \times 6\) coefficient matrix; if the absorbers are not present, then the system collapses to a \(4 \times 4\) system of the same form with the vector on the right simply \(\text{col}(0, 1, 0, 0)\).

Once the \(a\)'s and \(b\)'s are known, the amplitude of the displacement for each component is simply

\[ \text{displ ampl} = \sqrt{a^2 + b^2} \]

There is a second question of concern regarding these motions and this is, “what is the force transferred at through each of these viscoelastic couplings?” After the motions of the several points are known, that matter can be addressed as follows. Consider two points \(i\) and \(j\) moving with displacements \(x_i\) and \(x_j\) and joined by a
spring–damper combination (if the damper is absent, it will only simplify things; if one point does not move, that will also simplify things). Then,

\[
x_i = a_i \cos \Omega t + b_i \sin \Omega t \\
\dot{x}_i = -\Omega a_i \sin \Omega t + \Omega b_i \cos \Omega t
\]

\[
x_j = a_j \cos \Omega t + b_j \sin \Omega t \\
\dot{x}_j = -\Omega a_j \sin \Omega t + \Omega b_j \cos \Omega t
\]

The force through the coupling element is then

\[
F = K (x_j - x_i) + C (\dot{x}_j - \dot{x}_i)
\]

\[
= K (a_j \cos \Omega t + b_j \sin \Omega t - a_i \cos \Omega t - b_i \sin \Omega t)
+ C (-\Omega a_j \sin \Omega t + \Omega b_j \cos \Omega t + \Omega a_i \sin \Omega t - \Omega b_i \cos \Omega t)
\]

\[
= K [(a_j - a_i) \cos \Omega t + (b_j - b_i) \sin \Omega t]
+ C \Omega [(b_j - b_i) \cos \Omega t - (a_j - a_i) \sin \Omega t]
\]

\[
= K \left[(a_j - a_i)^2 + (b_j - b_i)^2\right]^{1/2} \left(\cos \lambda \cos \Omega t + \sin \lambda \sin \Omega t\right)
+ C \Omega \left[(a_j - a_i)^2 + (b_j - b_i)^2\right]^{1/2} \left(\sin \lambda \cos \Omega t - \cos \lambda \sin \Omega t\right)
\]

where

\[
\cos \lambda = \frac{a_j - a_i}{\sqrt{(a_j - a_i)^2 + (b_j - b_i)^2}^{1/2}} \\
\sin \lambda = \frac{b_j - b_i}{\sqrt{(a_j - a_i)^2 + (b_j - b_i)^2}^{1/2}}
\]

Continuing then,
\[ F = K \left[ (a_j - a_i)^2 + (b_j - b_i)^2 \right]^{1/2} \cos(\Omega t - \lambda) \\
- C\Omega \left[ (a_j - a_i)^2 + (b_j - b_i)^2 \right]^{1/2} \sin(\Omega t - \lambda) \\
= \left[ (a_j - a_i)^2 + (b_j - b_i)^2 \right]^{1/2} \left[ K \cos(\Omega t - \lambda) - C\Omega \sin(\Omega t - \lambda) \right] \\
= \left[ (a_j - a_i)^2 + (b_j - b_i)^2 \right]^{1/2} \left( K^2 + C^2 \Omega^2 \right)^{1/2} \left[ \cos \gamma \cos(\Omega t - \lambda) - \sin \gamma \sin(\Omega t - \lambda) \right] \\
= \left[ (a_j - a_i)^2 + (b_j - b_i)^2 \right]^{1/2} \left( K^2 + C^2 \Omega^2 \right)^{1/2} \cos(\Omega t + \gamma - \lambda) \\
\]

where

\[ \cos \gamma = \frac{K}{(K^2 + C^2 \Omega^2)^{1/2}} \]
\[ \sin \gamma = \frac{C\Omega}{(K^2 + C^2 \Omega^2)^{1/2}} \]

Finally, the magnitude of the force through the connection is just

\[ |F| = \left[ (a_j - a_i)^2 + (b_j - b_i)^2 \right]^{1/2} \left( K^2 + C^2 \Omega^2 \right)^{1/2} \]

This is the basis for calculating the force through all of the connections in the system.

Without the absorber in place, the system response is as shown in Figures 3 and 4, where \( f_{\text{req}} \) represents shaft speed \( \Omega \) expressed in Hz.

When the vibration absorbers are added to the system, the system response are changed to those shown in Figures 5 and 6.

There are several important points to notice regarding the application of the Frahm damper (technically the Frahm absorber):

1. The motion of the primary object is completely stopped at the frequency of interest (running speed for the machine). This is shown by the fact that both the displacement and force amplitudes are zero at the running frequency.

2. At the running speed, the forces through the secondary coupling, \( F_{12} \) and the absorber coupling, \( F_{13} \) are in fact equal but exactly out of phase. The equality of magnitudes is evident in Figure 6, although there is nothing there to show that they are out of phase. Figure 5 shows that the amplitude of motion on the absorber is quite high at the running speed. This is not surprising, but it means that fatigue must be considered seriously in this design.
Figure 3: Displacement Response Without Absorbers (Upper Curve is $X_1$)
3. This is clearly a “tuned system” and is designed to function at a specific frequency. At frequencies other than the design point, the results may very well be worse than if the absorber were not present.

The last point is of considerable importance in terms of choosing applications where the Frahm absorber may be applied. The power plant fan in this application is just about ideal because the fan speed will always be very close to the design speed, 890 rpm. Utility generator shafts are another place where shaft speed is very constant and Frahm absorbers can be used well.
Figure 5: Displacement Response Curves With Absorbers In Place
Figure 6: System Force Amplitudes With Absorbers In Place