Motion in a Moving Coordinate System: An Actuated Deck Plate on a Navy Ship

1 Introduction

Consider a flat plat, normally at rest embedded in the deck of a ship, but capable of being tilted up on one edge by means of a hydraulic actuator. A possible application might be the jet blast deflector on the deck of an aircraft carrier, but there are other possibilities as well. The whole situation is shown in Figure 1. The purpose for this analysis would probably be for the design of the control system for the actuator that will elevate the plate.

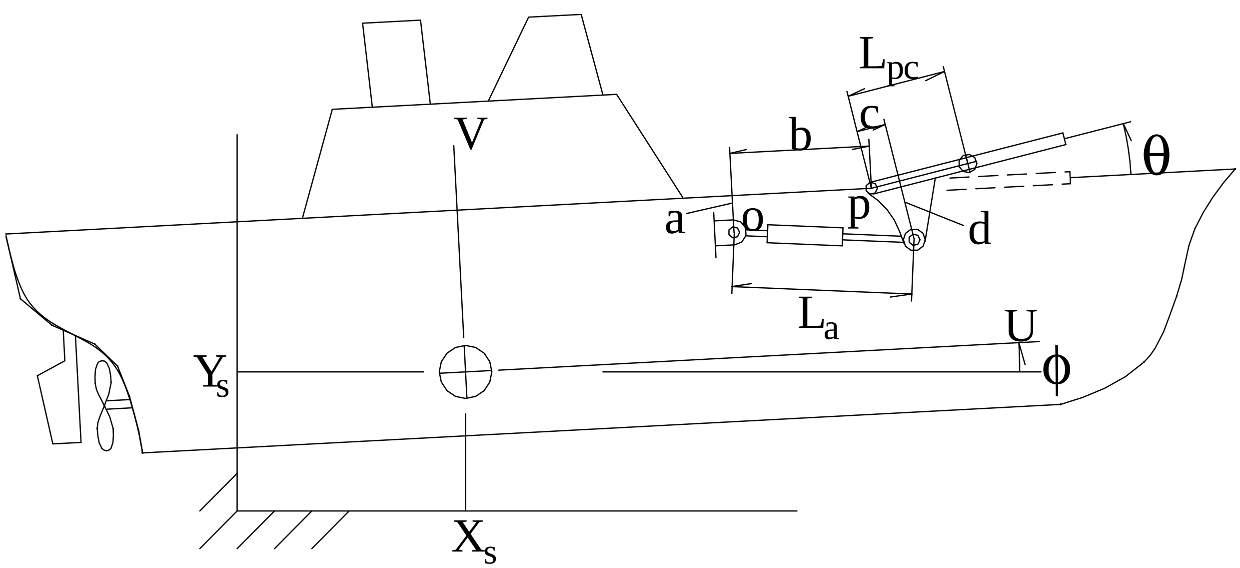

Figure 1: Plate in Ship's Deck

As shown in Figure 1, the plate is embedded in the foredeck of the ship, with a hinge at point p, and the elevation of the plate above the deck described by the angle θ. The plate center of mass is located at the distance L_{pc} away from the hinge pin. The actuation is accomplished through an arm extending below the plate a distance d at a distance c away from the hinge pin. The hydraulic actuator cylinder extends from this arm back to a point o fixed on the ships structure at a location a below the deck and distance b aft of the hinge pin.

Now the ship itself is a moving platform, moving with respect to an inertial coordinate system fixed on the earth. The ship center of mass motion is described by (X_s, Y_s), and the pitching motion of the ship is described by the angle ϕ. In actual fact, the value of X_s is virtually meaningless because the horizontal distance from some fixed reference is, in most cases, of no value and usually is unknown. Similarly, the vertical distance is of no importance, because the ship always floats on the surface, no matter how deep the water may be. However, their derivatives may be very much matters of interest, and this is why is is useful to begin by defining the coordinates themselves, in order to get the proper relations with the other variables. Similarly, the angle ϕ is always small, never more than a few degrees for a large ship, but its derivatives can be fairly large. Thus all of these matters are taken into account in developing the dynamics of the deck plate.

In truth, $X_s(t), Y_s(t)$ and $\phi(t)$ can never be known as inputs to a system simulation. The forward motion of the ship is comprised of two terms, the mean forward velocity and also the *surge*, or unsteady term. Obviously it is the surge that contributes to $\ddot{X}_s$, although both components are found in $\dot{X}_s$. The vertical motion is called *heave*, and this is what gives rise to both $\dot{Y}_s$ and $\ddot{Y}_s$. The in–plane rotation of the ship is called *pitch*, which gives ϕ and all of its derivatives. Information for estimating the magnitude of these several motions on various types of US warships in different sea states may be found in MIL – STD – 1399[1]. This information only provides an estimate of the amplitudes of each effect, without providing any phase information. Therefore it cannot be used to provide a true, time function description of these inputs. There may be other, better data as well.

2 Kinematics

There are two aspects of the kinematic analysis. The first part is the analysis required to get the velocity components required in order to be able to write the kinetic energy as required for the application of the Lagrange equation of motion. Secondly, it will be necessary to describe the work of the hydraulic actuator, and this means being able to calculate the length of the actuator, $L_a(\theta)$, along with the derivative, $dL_a/d\theta$. These two matters are taken up separately below.

2.1 Plate Center of Mass Motion

The location of the hinge pin, p, in the ship body coordinate system (U, V) is assumed to be (u_p, v_p), a known point. The first task then is to find global coordinates for the hinge pin.

$$
\begin{aligned}
x_p &= X_s + u_p \cos\phi - v_p \sin\phi \\
y_p &= Y_s + u_p \sin\phi + v_p \cos\phi
\end{aligned}
$$

[1] DOD-STD-1399(NAVY), SECTION 301A, 21 July 1986 SUPERSEDING MIL-STD-1399(NAVY) SECTION 301 17 February 1972 (See 6.4)

From this point, it is a short step to get to the global coordinates for the plate center of mass by simply adding the transfer terms,

$$
\begin{aligned}
x_{pc} &= X_s + u_p \cos\phi - v_p \sin\phi + L_{pc} \cos(\theta + \phi) \\
y_{pc} &= Y_s + u_p \sin\phi + v_p \cos\phi + L_{pc} \sin(\theta + \phi)
\end{aligned}
$$

These last expressions are differentiated with respect to time to obtain the components of velocity for the plate:

$$
\begin{aligned}
\dot{x}_{pc} &= \dot{X}_s - \dot{\phi}(u_p \sin\phi + v_p \cos\phi) - \left(\dot{\theta} + \dot{\phi}\right) L_{pc} \sin(\theta + \phi) \\
\dot{y}_{pc} &= \dot{Y}_s + \dot{\phi}(u_p \cos\phi - v_p \sin\phi) + \left(\dot{\theta} + \dot{\phi}\right) L_{pc} \cos(\theta + \phi)
\end{aligned}
$$

It is evident that the actuator driven motion, θ, is quite entangled with the several motions of the ship in these velocity expressions.

2.2 Actuator Length Calculation

Two loop equations can be written, parallel to the two ships body coordinate axes. These are

$$
\begin{aligned}
b + c\cos\theta + d\sin\theta - L_a \cos\beta &= 0 \\
a + c\sin\theta - d\cos\theta + L_a \sin\beta &= 0
\end{aligned}
$$

where β is the angle between the actuator axis and the $U-$ axis. If the angle β is eliminated by the evident squaring operations, there results an expression for the square of the actuator length,

$$
L_a^2 = 2(bc - ad)\cos\theta + 2(bd + ac)\sin\theta + a^2 + b^2 + c^2 + d^2
$$

Obviously, this is readily solved for L_a simply by taking a square root. This also serves as a point of departure for the calculation of the derivative, leading to

$$
\frac{dL_a}{d\theta} = \frac{1}{L_a}\left[-(bc - ad)\sin\theta + (bd + ac)\cos\theta\right]
$$

This result is needed below in the expression of the equation of motion.

3 System Dynamics

At this point, it is in order to begin to assemble the various parts that go into the equation of motion. These include the kinetic energy with the appropriate derivatives, the potential energy and the required derivative, and the nonconservative virtual work from which the nonconservative generalized force is obtained.

3.1 Kinetic Energy

The system kinetic energy is comprised simply of the energy of motion of the moving plate (neglecting the moving mass of the actuator cylinders), so that is written as

$$T = \frac{1}{2} M \left(\dot{x}_{pc}^2 + \dot{y}_{pc}^2 \right) + \frac{1}{2} I_c \left(\dot{\theta} + \dot{\phi} \right)^2$$

Direct substitution of the results obtained above is clearly going to be rather messy, so this is a place where a computer algebra program becomes particularly advantageous. After considerable simplification, this becomes

$$\begin{aligned} T &= \{ (u_p \cos\theta + v_p \sin\theta) \dot{\phi}^2 \\ &+ \left[(u_p \cos\theta + v_p \sin\theta) \dot{\theta} - \dot{X}_s \sin(\theta + \phi) + \dot{Y}_s \cos(\theta + \phi) \right] \dot{\phi} \\ &+ \left[-\dot{X}_s \sin(\theta + \phi) + \dot{Y} \cos(\theta + \phi) \right] \dot{\theta} \} M L_{pc} \\ &+ \{ \frac{1}{2} \left(u_p^2 + v_p^2 \right) \dot{\phi}^2 + \frac{1}{2} \left(\dot{X}_s^2 + \dot{Y}_s^2 \right) \\ &+ \left[\left(\dot{Y}_s u_p - \dot{X}_s v_p \right) \cos\phi - \left(\dot{Y}_s v_p + \dot{X}_s u_p \right) \sin\phi \right] \dot{\phi} \} M \\ &+ \frac{1}{2} \left(\dot{\theta} + \dot{\phi} \right)^2 I_p \end{aligned}$$

where use has been made of the parallel axis theorem, $I_p = I_c + M L_{pc}^2$. As noted previous, the ship motion with that of the deck plate is truly complicated!

In preparation for the application of the Lagrange equation, there are several derivatives of the kinetic energy that are required. Because of the complexity of the individual expressions, they are not all developed in detail here, but rather the final result only is given. A computer algebra program is again a great aid in this effort.

$$\begin{aligned} \frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}} - \frac{\partial T}{\partial \theta} &= [\ddot{Y}_s \cos(\theta + \phi) - \ddot{X}_s \sin(\theta + \phi) \\ &+ (u_p \cos\theta + v_p \sin\theta) \ddot{\phi} + (u_p \sin\theta - v_p \cos\theta) \dot{\phi}^2] M L_{pc} \\ &+ \left(\ddot{\theta} + \ddot{\phi} \right) I_p \end{aligned}$$

3.2 Potential Energy

The easiest way to include the gravitational effects (the weight of the plate) is to make use of a gravitational potential energy. To that end then, the potential energy is then

$$\begin{aligned} V &= M\,g\,y_{pc} \\ &= M\,g\left[Y_s + u_p \sin\phi + v_p \cos\phi + L_{pc} \sin\left(\theta + \phi\right)\right] \end{aligned}$$

The actual value of the potential energy is of no importance in itself, but rather only the derivative with respect to θ, thus:

$$\frac{\partial V}{\partial \theta} = MgL_p \cos\left(\theta + \phi\right)$$

3.3 Nonconservative Generalized Force

The actuator force, F_a, is necessarily a nonconservative force that puts energy into the system because F_a is positive when it is extending the length L_a. There could be other nonconservative loads to be considered such as wind loading and friction, but those are neglected for this analysis. The virtual work of the nonconservative force is

$$\begin{aligned} \delta W^{NC} &= F_a \delta L_a \\ &= F_a K_a \delta\theta \end{aligned}$$

From this, the nonconservative generalized force is

$$\begin{aligned} Q^{nc} &= F_a K_a \\ &= \frac{F_a}{L_a}\left[-\left(bc - ad\right)\sin\theta + \left(bd + ac\right)\cos\theta\right] \end{aligned}$$

3.4 Equation of Motion

The Lagrange equation of motion is of the form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\theta}} - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = Q^{nc}$$

and all of the pieces have been developed individually. All that remains is to complete the assembly of the equation of motion.

$$
\begin{aligned}
&[\ddot{Y}_s \cos(\theta+\phi) - \ddot{X}_s \sin(\theta+\phi) \\
&+ (u_p \cos\theta + v_p \sin\theta)\,\ddot{\phi} + (u_p \sin\theta - v_p \cos\theta)\,\dot{\phi}^2]ML_{pc} \\
&+ \left(\ddot{\theta} + \ddot{\phi}\right) I_p + MgL_p \cos(\theta+\phi) = \frac{F_a}{L_a}\left[-(bc-ad)\sin\theta + (bd+ac)\cos\theta\right]
\end{aligned}
$$

This can be seen as the differential equation for the motion $\theta(t)$, where $X_s(t)$, $Y_s(t)$, $\phi(t)$, and $F_a(t)$ are all considered as inputs to that motion. It is, of course, quite evident that $\theta(t)$ is badly tangled with the various inputs in this equation, and the only hope for a solution is by numerical means, but that would require that $X_s(t)$, $Y_s(t)$, and $\phi(t)$ could be known, which they cannot be.

This is the point where this analysis ends as the purpose was simply to obtain the equation of motion, showing the way in which all of these several quantities affect the motion. This has been accomplished.